Now for sure or later with a risk? Modeling risky inter-temporal choice as accumulated preference

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Abstract

Research on risky and inter–temporal decision–making often focuses on descriptive models of choice. This class of models sometimes lack a psychological process account of how different cognitive processes give rise to choice behavior. Here, we attempt to decompose these processes using sequential accumulator modeling (i.e., the Linear Ballistic Accumulator model; LBA). Participants were presented with pairs of gambles that either involve different levels of probability or delay (Experiment 1) or a combination of these dimensions (both probability and delay; Experiment 2). Response times (RTs) were recorded as a measure of preferential strength. We then combined choice data and response times, and utilized variants of LBA to explore different assumptions about how preferences are formed. Specifically we show that a model which allows for the subjective evaluation of a fixed now/certain option to change as a function of the delayed/risky option with which it is paired provides the best account of the combined choice and RT data. The work highlights the advantages of using cognitive process models in risky and inter–temporal choice, and points towards a common framework for understanding how people evaluate time and probability.

Keywords: Inter–temporal Choice; Risky Choice; Evidence Accumulation; Cognitive Modeling; Linear Ballistic Accumulator

Introduction

You have just won the lottery and the prize is $10,000. Do you use your money now, or do you put it in a bank account, for one year, and then take out $11,500? This choice is an example of an *inter–temporal choice*, it involves tradeoffs between sooner–smaller (SS) and larger–later (LL) options. Consider a second dilemma. You are a contestant in "Who wants be to millionaire?": you can either choose to go home with $10,000, or you can choose to answer one more question. If you get it right, you get $20,000, but if you get it wrong, you go home with nothing. The second choice is an example of a *risky choice*, it involves tradeoffs between certain and risky but more valuable options. Most studies on inter–temporal and risky choice have employed context-free monetary choice dyads between SS and LL options on the one hand, for example, a choice between $10 now or $15 in two months (e.g., Chapman & Weber, 2006; Loewenstein & Prelec, 1992), and between certain and risky options on the other hand, for example, a choice between $30 for sure or $40 with 80% chance or nothing otherwise (e.g., Kahneman & Tversky, 1979).

Two hallmarks of traditional research on inter–temporal and risky choice are i) examination of the two types of choice in isolation, and ii) evaluation of preferences in terms of their coherence (or lack thereof) with normative economic principles. This large body of work has revealed key insights into the types of factors that affect risky or inter-temporal choice, but "the interaction between risk and delay is complex and not easily understood" (B. J. Weber & Chapman, 2005, p. 104).

In the domain of inter–temporal choice, the dominant approach is to examine whether choices across time adhere to Discounted Utility Theory (DUT; Samuelson, 1937). DUT implies that decision makers maximize a weighted sum of utilities with exponentially declining discount weights. In the domain of risky choice, research has focused on Expected Utility Theory (EUT). EUT views decision makers as maximizing a weighted sum of utilities with their probabilities of occurrence (e.g., Epper, Fehr-Duda, & Bruhin, 2011; Prelec & Loewenstein, 1991).

DUT and EUT are normative models of choice; they provide principles according to which rational decision makers should behave (Newell, Lagnado, & Shanks, 2015). However, extensive research has documented several violations of these principles (e.g., Allais, 1953; Thaler, 1981). The standard approach to account for these violations is to modify the theories but to retain their core constituents. Thus for inter–temporal choice, hyperbolic functions which allow decreasing discount rates rather than constant (i.e., exponential) rates are used to capture observed choice "anomalies" (e.g., Green & Myerson, 2004). For risky choice, allowing a non–linear probability weighting function provides explanations of commonly observed behavioral effects and preference reversals (e.g., Kahneman & Tversky, 1979; Tversky & Kahneman, 1992).

These modified models, such as Prospect Theory (Kahneman & Tversky, 1979) are descriptive: they provide a description not of how decision makers should behave but how

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they appear to behave when confronted with such choices (at least at the aggregate level, cf. Epper et al., 2011). Such models (e.g., Cumulative Prospect Theory for risky choice and Hyperbolic Discounting for inter–temporal choice) are utility–based models: a utility (or subjective value) is calculated for each option, and the option with the highest utility is chosen. However, what these models lack is a psychological process account of why choices are better fit by hyperbolic than exponential functions, or by non–linear than linear weighting functions (cf. Stewart, Chater, & Brown, 2006). In other words, these models do not explain why and/or how the utility of each option is estimated and the psychological processes that are involved. To answer this 'how/why' question requires the development of *cognitive process models* which specify the components and relations between the (thought) processes engaged when making such choices (e.g., Appelt, Hardisty, & Weber, 2011; Brandstätter, Gigerenzer, & Hertwig, 2006; Shafir, Simonson, & Tversky, 1993; Vlaev, Chater, Stewart, & Brown, 2011; E. U. Weber et al., 2007).

In the field of speeded multi–alternative forced choice decision–making, such cognitive process models have been in use for almost four decades (e.g., Ratcliff, 1978; Brown & Heathcote, 2008). The cognitive models of choice in the field of response time (RT) research are called sequential accumulator models. Among others, these models have been successfully applied to experiments on perceptual discrimination, letter identification, lexical decision, categorization, recognition memory, and signal detection (e.g., Ratcliff, 1978; Ratcliff, Gomez, & McKoon, 2004; Ratcliff, Thapar, & McKoon, 2006, 2010; van Ravenzwaaij, Dutilh, & Wagenmakers, 2012; van Ravenzwaaij, van der Maas, & Wagenmakers, 2011; Wagenmakers, Ratcliff, Gomez, & McKoon, 2008). Evidence accumulation models such as decision field theory (DFT; Busemeyer & Townsend, 1993) and the leaky competing accumulator (LCA; Usher & McClelland, 2001) have been applied in the domain of risky choice.

One of the advantages of such a modeling approach is that it allows researchers to decompose observed RTs and choice proportions into latent psychological processes such as speed of cognitive processing, response caution, and non–decision time. More traditional analyses make no attempt to explain the observed data by means of a psychologically plausible process model.

One key difference between inter–temporal and risky choice on the one hand and traditional RT research on the other is that in the former field, decisions are rarely timed (but see e.g., Dai & Busemeyer, 2014; Rodriguez, Turner, & McClure, 2014). This is a shame, as RTs could potentially teach us a lot about people's preferences. For instance, consider on the one hand the choice between receiving $100 now or $10,000 in two months and on the other hand the choice between receiving $100 now or $150 in two months. In both instances, you might prefer the larger–later option. As a result, your choice preference will look the same. However, the first choice was most likely a much easier one to make than the second one. Your *strength of preference* in the first choice in favor of the larger–later option was likely much larger. This strength of preference would likely be reflected in a lower RT compared to the RT associated with your decision for the second choice.

Another difference between inter–temporal and risky choice on the one hand and traditional RT research on the other is that the response options are about preference and as such, there are no correct answers. This presents a difference on the conceptual level, but on the model level, there are no obstacles as evident by some recent applications of sequential accumulator models to preference data (see e.g., Dai & Busemeyer, 2014; Hawkins et al., 2014; Rodriguez et al., 2014; Trueblood, Brown, & Heathcote, 2014).

In this paper, we present data from two experiments that contained both inter–temporal and risky choices. In Experiment 1 participants had to make a choice between a small-sooner ("now") and a larger-later option for the inter–temporal choice trials, and a choice between a certain-small and a risky-larger option for the risky choice trials. In Experiment 2 the inter–temporal and risky components were combined within trials: participants had to make a choice between a certain-now-small and a risky-later-larger option. In both experiments the now/certain options remained constant across choice trials: $100 with certainty, right now. We pressed our participants to make their responses as quickly as possible while still being accurate. We then applied a cognitive process model to the data.

One of the main objectives of the present cognitive process model analysis is to uncover the potential for a shared component that drives decision–making in both inter–temporal and risky choices. Previous research has identified several parallels and similarities between probability and delay. For example, Chapman and Weber (2006) examined whether two well-documented biases in risky and inter–temporal choice (the *common ratio* effect, and the *common difference* effect, respectively) can be accounted for by the same underlying mechanism. Other studies have found evidence for psychological equivalence between probability and delay, suggesting that probability can be translated or treated as delay (e.g., Rachlin, Raineri, & Cross, 1991; Yi, de la Piedad, & Bickel, 2006), and vice-versa, that delay can be translated into probability or uncertainty (Baucells & Heukamp, 2010; Keren & Roelofsma, 1995). In addition, recent theoretical and modeling attempts have assumed similar functional forms for delay discounting (i.e., decrease of a reward value with increases in delay) and probability discounting (i.e., decrease of a reward value with decreases in probability). For instance, Vanderveldt, Green, and Myerson (2015) observed that a hyperboloid function of delay and probability discounting can describe discounting patterns in both domains. While their hyperboloid model provides an excellent fit to the choice data, it is a descriptive "as–if" model, in the sense that it does not account for the underlying thought processes that drive preferential choice. With our cognitive process model and the simultaneous examination of choices and RTs (i.e., strength of preference), we take the idea of parallelism in delay and probability discounting one step further by exploring the potential for a unifying psychological process that governs preferences in both inter–temporal and risky choices.

The model also allowed us to test to what extent the absolute attractiveness of the now/certain options (which was always an immediate $100 with certainty) change with variations in the delayed/risky choice options with which it was paired. Such a test is difficult with behavioral data or descriptive models of choice that typically only provide insight into the relative attractiveness of two presented choice options. Following this, our model constitutes a natural test of integrative expected utility-based models which assume an overall fixed value for each option irrespective of the context or the alternative to which it is compared (see also Brandstätter et al., 2006).

The remainder of this paper is organized as follows. In the next section, we will introduce our sequential accumulator model of choice: the Linear Ballistic Accumulator model (LBA; Brown & Heathcote, 2008). We will then describe our experiments in detail. Next, we discuss first the behavioral results and then the modeling results of the data. We conclude with a discussion of the gains of such a cognitive modeling approach and lessons to be learned about the shared nature of delay and probability.

The Linear Ballistic Accumulator model

In the LBA for multi–alternative RT tasks (Brown & Heathcote, 2008), the decision–making process is conceptualized as the accumulation of information over time. A response is initiated when the accumulated evidence reaches a predefined threshold. An illustration for an inter–temporal choice with two response options is given in Figure 1.

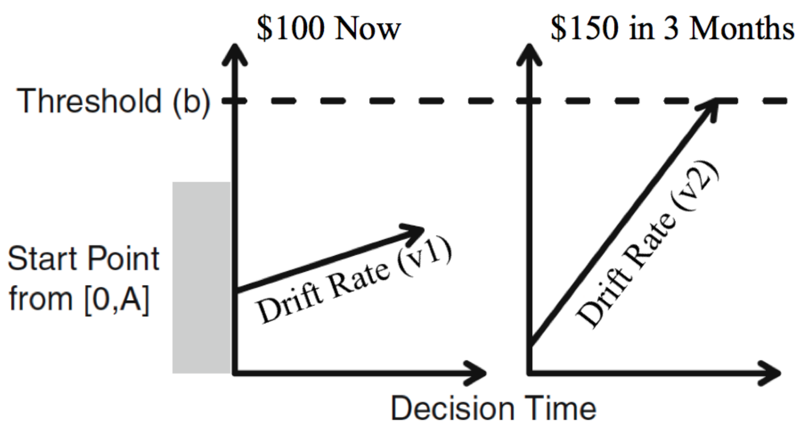

Figure 1. The LBA and its parameters for an inter–temporal choice with two response options. Evidence accumulation begins at start point k, drawn randomly from a uniform distribution with interval $[0, A]$. Evidence accumulation is governed by drift rate d, drawn across trials from a normal distribution with mean ν and standard deviation s. A response is given as soon as one accumulator reaches threshold b. Observed RT is an additive combination of the time during which evidence is accumulated and non–decision time $t0$.

The LBA assumes that the decision process starts from a random point between 0 and A, after which information is accumulated linearly for each response option. The rate of this evidence accumulation is determined by drift rates d_1 and d_2, normally distributed over trials with means ν_1 and ν_2, and standard deviation s, which we assume here to be equal for both accumulators. For this application, drift rates are truncated at zero to prevent negative accumulation rates. Threshold b determines the speed–accuracy tradeoff; lowering b leads to faster RTs at the cost of a higher error rate (but see Rae, Heathcote, Donkin, Averell, & Brown, 2014).

Together, these parameters generate a distribution of decision times DT. The observed RT, however, also consists of stimulus–nonspecific components such as stimulus encoding, response preparation and motor execution, which together make up non–decision time $t0$. The model assumes that $t0$ simply shifts the distribution of DT, such that $RT = DT + t_0$ (Luce, 1986). Hence, the three key components of the LBA are (1) the speed of information processing, quantified by mean drift rate ν; (2) response caution, quantified by distance from start point to threshold that averages at $b - A/2$; and (3) non–decision time, quantified by $t0$.

The LBA has been applied to a number of perceptual discrimination paradigms (e.g., Cassey, Gaut, Steyvers, & Brown, 2015; Cassey, Heathcote, & Brown, 2014; Forstmann et al., 2008, 2010; Ho, Brown, & Serences, 2009; van Ravenzwaaij, Provost, & Brown, in press). Recently, the LBA has also been applied to preference data. For instance, Hawkins et al. (2014) applied the LBA to consumer preference data toward mobile phones. In an adaptation of the LBA developed by Trueblood et al. (2014), the model was fit to preference data of three kinds of context effects: similarity, compromise, and attraction. Rodriguez et al. (2014) applied the LBA to inter–temporal choice data and concluded that "perceptual and value–based decision–making may depend on similar comparison and selection processes" (p. 7).

The interpretation of the drift rate parameter changes when applying sequential accumulator models to preference data without an inherent correct answer. Rather than speed of information processing, drift rate reflects the strength of preference for a choice option. For this application, we define drift rates as representing a weighted sum of an option's attribute values (amount, delay, and probability). In other words, each attribute's contribution to the strength of preference depends on the importance or attention placed to each attribute, quantified by scaling parameters (or weights; see the Model Implementation section for more details). This definition of preferential strength allows us to test three specific accounts of how choice preferences vary with different levels of delay and probability.

Specifically, we examine how preferences for the now/certain options are formed in relation to the values of the choice attributes of the delayed/risky options. The first account ("independent") assumes that the value of the now/certain option changes "independently" with different alternatives for the delayed/risky option. The choice attributes (amount, delay, and probability) in the now/certain and delayed/risky options have different weights, suggesting that the importance or attention placed on each attribute varies between the two options. On the other hand, the "invariant" account simply assumes that the absolute value (preferential strength) of the now/certain option remains constant across all choice trials, irrespective of the attribute values of the delayed/risky option. Consequently, this model suggests that a single absolute value for the now/certain option is estimated (that is, a single drift rate across all trials – no scaling parameters or weights needed). This "invariant" account resembles expected utility-based models, which assume a single fixed value for an option regardless of the context (i.e., alternative options) in which a decision is made. The last account ("symmetrical") presents a compromise between the two aforementioned "extreme" accounts: while the value of the now/certain option does not remain constant, the choice attribute weights are identical between the two options. We formally describe these variants of the LBA in the Model Implementation section below.

In this paper, we fit the LBA to inter–temporal and risky choice data simultaneously. Thus, our work presents the first attempt to examine the potential for a unifying underlying process that governs preferences in both inter–temporal and risky choices and to decompose the relative importance of risk and delay in people's choices.

Experiment 1

We set out to model people's preferences on inter–temporal and risky choices separately. Participants completed a task with two separate blocks of inter–temporal choice and risky choice trials. We report the behavioral results of this experiment, as well as the modeling results provided by fitting the LBA.

Method

Participants. Forty undergraduate students (26 female; $M_{age} = 19.40$) at the University of New South Wales participated in return for course credit. For each participant, one of their preferences from the risky choice trials was randomly selected. If the participant preferred the risky option in that specific choice trial, the gamble was played for real (e.g., \$200 with 50% chance). In case of a win, the participant was paid 2% of the amount (e.g., \$4 as 2% of \$200) and nothing otherwise. Those who preferred the sure option were paid \$2 (i.e., 2% of \$100 for sure)[1].

Materials. The experiment consisted of 380 inter–temporal choice and 380 risky choice trials. For the inter–temporal choices, participants had to indicate what they preferred: \$100 now or \$$X$ in D months, with \$$X$ varying from \$120 to \$500 in \$20 increments (for a total of 20 amounts) and D varying from 2 months to 38 months in 2 month increments (for a total of 19 delays). Thus, every combination of amount and delay was presented to participants once as an alternative to \$100 now ($D = 0$).

For the risky choices, participants had to indicate what they preferred: \$100 for sure or \$$X$ with P% chance, with \$$X$ varying from \$120 to \$500 in \$20 increments (for a total of 20 amounts) and P varying from 5% to 95% in 5% increments (for a total of 19 probabilities). Thus, every combination of amount and probability was presented to participants once as an alternative to \$100 for sure ($P = 100\%$).

Procedure. Participants completed the experiment in two sessions, of 380 trials each (190 risky choice trials, and 190 inter–temporal choice trials). Within a session the order of the trials was blocked (i.e., all risky together, all inter–temporal together) and counterbalanced. The two sessions were separated by a minimum of three hours. (i.e., some participants completed the sessions on consecutive days, others in the morning and afternoon of the same day).

Implementation of the Model

We used a hierarchical Bayesian implementation of the LBA (Turner, Sederberg, Brown, & Steyvers, 2013). Advantages of the hierarchical Bayesian framework include the ability to fit the LBA to data with relatively few trials, because the model borrows strength from the hierarchical structure. This advantage is important, as we are working with a task for which we essentially have only a single trial per participant for each type of choice (one single combination of \$$X$ and D for inter–temporal choices and a combination of \$$X$ and P% for risky choices). The Bayesian set–up allows for using Markov chain Monte-Carlo (MCMC) sampling, which is an efficient way of finding the optimal set of parameters (Gamerman & Lopes, 2006; Gilks, Richardson, & Spiegelhalter, 1996; van Ravenzwaaij, Cassey, & Brown, in press).

[1] At the outset, participants were told that one trial from the experiment would be selected and the gamble would be played for real. To facilitate payment, the (pseudo)randomly selected trial always came from the risky choice trials, but participants did not know this, thus creating the impression of equal incentives for both phases of the experiment, without involving any deception.

We found that the best fit was achieved with a version of the LBA that fit the following parameters: two parameters for the upper range of starting point A (A_D for inter–temporal and A_R for risky choices), two parameters for threshold b (b_D for inter–temporal and b_R for risky choices), and one non–decision time parameter $t0$. We fit three different models that differed on how they model the choice process. We first describe the "independent" model, then describe the two model variations by referring to changes to the "independent" model.

For the inter–temporal choice task, the "independent" model included drift rates for the "now" and "delayed" options as follows:

$$\begin{aligned} \nu_N &= \nu_{N_0} - \alpha_{N_X} \times (\$X/20 - 6) - \alpha_{N_D} \times (19 - D/2) \\ \nu_D &= \nu_{D_0} - \alpha_{D_X} \times (25 - \$X/20) - \alpha_{D_D} \times (D/2 - 1) \end{aligned} \tag{1}$$

where ν_N and ν_D denote drift rates for the now and the delayed choice options, ν_{N_0} and ν_{D_0} denote offset parameters for the now and the delayed choice options, X denotes the amount in dollars for the delayed choice option, D denotes delay in months for the delayed choice option, α_{N_X} and α_{D_X} are amount scale parameters for the now and the delayed choice options, and α_{N_D} and α_{D_D} are delay scale parameters for the now and the delayed choice options. $\nu_N = \nu_{N_0}$ if $\$X = 120$ and $D = 38$ (the option that most favors the "now" choice). $\nu_D = \nu_{D_0}$ if $\$X = 500$ and $D = 2$ (the option that most favors the "delayed" choice). This results in a total of 9 parameters to be estimated: A_D, b_D, $t0$, ν_{N_0}, α_{N_X}, α_{N_D}, ν_{D_0}, α_{D_X}, and α_{D_D}.

For the risky choice task, the "independent" model included drift rates for the "certain" and "risky" options as follows:

$$\begin{aligned} \nu_C &= \nu_{C_0} - \alpha_{C_X} \times (\$X/20 - 6) - \alpha_{C_P} \times (P/5 - 1) \\ \nu_R &= \nu_{R_0} - \alpha_{R_X} \times (25 - \$X/20) - \alpha_{R_P} \times (19 - P/5) \end{aligned} \tag{2}$$

where ν_C and ν_R denote drift rates for the certain and risky choice options, ν_{C_0} and ν_{R_0} denote offset parameters for the certain and risky choice options, X denotes the amount in dollars for the risky choice option, P denotes the payout probability for the risky choice option, α_{C_X} and α_{R_X} are amount scale parameters for the certain and the risky choice options, and α_{C_P} and α_{R_P} are probability scale parameters for the certain and the risky choice options. $\nu_C = \nu_{C_0}$ if $\$X = 120$ and $P\% = 5$ (the option that most favors the certain option). $\nu_R = \nu_{R_0}$ if $\$X = 500$ and $P\% = 95$ (the option that most favors the risky option). This results in a total of 8 extra parameters to be estimated: A_R, b_R, ν_{C_0}, α_{C_X}, α_{C_P}, ν_{R_0}, α_{R_X}, and α_{R_P}.[2] Thus, the "independent" model has 17 parameters to be estimated, which together should account for the distribution of response times and choice proportions for both inter–temporal and risky choice data.

We fit two other models that test specific assumptions about the underlying choice process. The first model ("invariant"), estimates a single ν_N parameter (i.e., drift rate for the now option) for all inter–temporal choice trials and a single ν_C parameter (i.e., drift rate for the certain option) for all risky choice trials. Thus, it replaces 6 free parameters (ν_{N_0}, α_{N_X}, α_{N_D}, ν_{C_0}, α_{C_X}, and α_{C_P}) with 2 free parameters. Conceptually, this simpler model assumes that the absolute value of the now/certain option does not change with different alternatives for the delayed/risky option. Essentially, the objectively invariant option would

[2]Note that $t0$ is fixed to be the same for the inter–temporal and the risky choice task.

also be perceived as invariant by the decision makers. This model has 13 free parameters to be estimated.

The final model ("symmetrical") presents a compromise between the "independent" model and the "invariant". The model assumes that $\alpha_{N_X} = \alpha_{D_X}$, $\alpha_{N_D} = \alpha_{D_D}$, $\alpha_{C_X} = \alpha_{R_X}$, and $\alpha_{C_P} = \alpha_{R_P}$. This means that contrary to the "invariant" model, the ν_N parameter and the ν_C parameter are not fixed to a single value. Instead, drift rates for the now/certain option vary symmetrically (though in the opposite direction) with drift rates for the delayed/risky option. This model has 13 free parameters to be estimated.

The comparison of the "independent", the "invariant", and the "symmetrical" models will teach us something about the change in subjective evaluation of the now/certain choice option. Is the subjective evaluation of the now/certain choice option fixed irrespective of the delayed/risky choice option, does the subjective evaluation of the now/certain choice option vary symmetrically with the delayed/risky choice option, or does the now/certain choice option vary independently with the delayed/risky choice option? We will use formal model comparison to find the account best supported by the data. Details on starting values, prior distributions, and number of iterations may be found in Appendix A.

We subdivide the results in two sections. The first subsection deals with the behavioral results, the second subsection deals with the modeling results.

Behavioral Results

Choice. All participants completed the experiment. Two participants were excluded from analysis: one participant had chosen the "delayed" option for every single choice, and another participant had seemingly responded randomly, producing responses that seemed largely inconsistent when compared against one another. Preference data for the inter-temporal choice trials can be found in the top-left panel of Figure 2. The figure shows group average proportion data. Proportions close to 0, displayed in blue, indicate a uniform preference for the \$100 now option. Proportions close to 1, displayed in yellow, indicate a uniform preference for the delayed option. The results show that participants prefer the now option when the delayed option does not pay very well (i.e., amounts not much higher than \$120) or when the delay is long (i.e., close to 38 months). In contrast, participants prefer the delayed option when the delayed option pays well (i.e., amounts close to \$500) or when the delay is short (i.e., close to now).

To examine the factors affecting choice of the smaller-sooner (SS; coded 0) or larger-later options (LL; coded 1) in the inter–temporal choice trials, we performed a mixed–effects logistic regression with amount and delay of the LL option as fixed effects (centered and scaled) and subject-specific random intercepts. As expected, there was a significant positive effect of amount ($b = 1.48, z = 43.00, p < .001$), indicating that as amount offered by the LL option increased, so did the likelihood of selecting the delayed option. In line with the observations from Figure 2, as delay increased participants were more likely to select the SS ("now") option ($b = -2.08, z = -51.20, p < .001$). In fact, in this particular set of gambles delay has a bigger effect than amount on determining choice.

Preference data for the risky choice trials can be found in the bottom-left panel of Figure 2. The figure shows group average proportion data. Proportions close to 0, displayed in blue, indicate a uniform preference for the risk-free option of \$100. Proportions close to 1, displayed in yellow, indicate a uniform preference for the risky option. The results

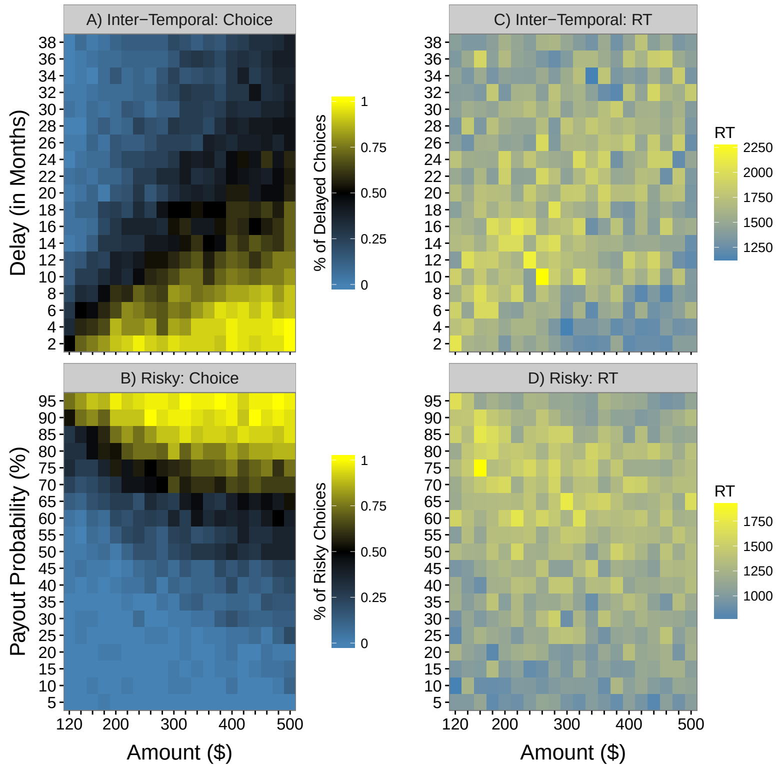

Figure 2. Behavioral data of the inter–temporal and risky choice trials in Experiment 1 averaged over participants. A) Proportion of preference data for inter–temporal choices. A proportion of 0 (blue) indicates exclusive preference for the $100 now option, a proportion of 1 (yellow) indicates exclusive preference for the delayed option. Black boxes represent proportions around 0.50. B) Proportion of preference data for risky choices. A proportion of 0 (blue) indicates exclusive preference for the $100 certain option, a proportion of 1 (yellow) indicates exclusive preference for the risky option. Black boxes represent proportions around 0.50. C) RT data for inter–temporal choices. D) RT data for risky choices. Low RTs are closer to blue on the color spectrum.

show that participants prefer the certain option when the risky option does not pay very well (i.e., amounts not much higher than $120) or when the payout probability is low (i.e., close to 5%). In contrast, participants prefer the risky option when the risky option pays well (i.e., amounts close to $500) or when the payout probability is high (i.e., close to 95%). Interestingly, we see that for this set of gambles payout probability appears to be more predictive than amount in determining participants' choices. For instance, when we examine the variation in preferences along the horizontal line where $P\% = 50$, we see preference shifts in the range of 0 to 0.60 (i.e., blue to black). When we examine the variation in preferences along the vertical line where amount $= 300$, we see preference shifts across the entire range of 0 to 1 (i.e., blue to yellow).

We performed the same analysis for the risky choices, with amount and payout probability of the risky option as fixed effects and subject-specific random intercepts. Consistent with the previous observations, payout probability ($b = 3.65, z = 52.63, p < .001$) is more predictive than amount ($b = 1.06, z = 29.51, p < .001$). The positive sign of both regression coefficients indicates that participants were more likely to select the risky option when amount and payout probability increased.

Response Times. The choice data indicates, perhaps unsurprisingly, that people prefer high payout, short delays, and high probability. What can we learn from the RT data? Before analyzing RT data, we excluded extreme responses following a two-stage procedure (see Hawkins, Hayes, & Heit, 2016). Specifically, we removed any responses slower than 10s or faster than 300ms (2.68% of all inter–temporal and risky trials). Next, for each participant, we removed those trials that were slower than 3 standard deviations from each participant's mean RT (total number of trials excluded = 4.88% of all trials).

Overall, higher RTs were associated with inter-temporal choices ($M = 1,574$ ms) compared to risky choices ($M = 1,227$ ms). Also, certain or now options were chosen faster ($M = 1,260$ ms) than risky or delayed options ($M = 1,648$ ms). Figure 2C shows group-average RT data for the inter-temporal choice trials. Low RTs are closer to blue on the color spectrum and high RTs are closer to yellow. The results show that the more extreme preferences in terms of proportion are accompanied by lower RTs (see also Dai & Busemeyer, 2014). The choices for which preferences varied among participants (~50%; the black diagonal in Figure 2A) are accompanied by higher RTs (the yellow diagonal in Figure 2C), indicating a lower absolute strength of preference. We analyzed RTs for the inter-temporal choices using a generalized linear mixed-effects model (with a Gamma distribution and non-transformed RTs) with amount and delay of the delayed option (scaled and centered), and choice response (now or delayed) as fixed effects and subject-specific random intercepts. While both amount and delay have a negative effect on RTs, only delay reached significance ($b_{amount} = -14.21, t = -1.39, p = .17$; $b_{delay} = -34.66, t = -3.11, p = .002$), indicating that as delay increases, we observe lower RTs. As expected, the choice between now or delayed options had an effect on RTs, with participants taking more time to select delayed options ($b = 100.30, t = 3.68, p < .001$).

Figure 2D shows group-average RT data for the risky choice trials. Similar to the inter-temporal choice RTs, the results show that the more extreme preferences in terms of proportion are accompanied by lower RTs. The choices for which preferences varied among participants (the darker axis from the top-left to the mid-right in Figure 2B) are accompanied by higher RTs (the yellow axis in Figure 2D), indicating a lower absolute strength of preference. The analysis (same as the one for the inter-temporal choices) revealed that the amount offered by the risky option did not have an effect on RTs ($b = -4.63, t = -0.55, p = .58$) whereas payout probability had a positive effect on RTs ($b = 77.54, t = 7.04, p < .001$), indicating that as payout probability increases RTs become higher. As for the inter–temporal choices, selections of the certain options are faster compared to selections of the risky options ($b = 74.94, t = 2.91, p = .004$).

In sum, people prefer high payout, short delays, and high probability. Within the parameter settings of our experiment (i.e., range of delays and amount), probability and delay seemed to guide people's preferences much more strongly than amount. As for response times, people take less time making risky choices than inter-temporal choices, and take a relatively short time to make choices for which response options are extreme. In the next section, we turn to the modeling results. We are looking for two things: 1) Are strengths of preference observed in the behavioral results reflected in the pattern of drift rates in the models we consider? 2) How do people weigh delay and probability in their choices?

Modeling Results

Parameter convergence was satisfactory. Numerically, we compare the "independent", the "invariant", and the "symmetrical" models by calculating the Deviance Information Criterion (DIC; Spiegelhalter, Best, Carlin, & van der Linde, 2002), a measure which balances goodness of fit against model complexity.[3] DIC values for the three models can be found in Table 1. The table shows that in terms of the DIC criterion, the best–fitting model is the "independent" model and the worst model is the "invariant" model. This suggests that decision makers' absolute valuation of the now and certain options in the inter–temporal and risky domains, respectively, change with different alternatives (different values of delay and probability) for the delayed and risky options.

Table 1
DICs for all three models fit to the data from Experiment 1.

Model	# Pars	DIC
Independent	17	124
Invariant	13	222
Symmetrical	13	144

Note: # Pars = Number of free parameters.

Moving to the estimated parameters, we examined posterior predictive data for the "independent" model, which are compared against the behavioral empirical data. It is important to note that every cell in our design contained only a single observation (i.e., any participant contributed only a single choice for each amount/delay or amount/probability combination). As such, our data are relatively noisy, as reflected by the width of the box–plots even on aggregated data, and the model fits reflect that noise (see Figure B1 in Appendix B). The model underestimates response times for the slower responses, but otherwise fits the data well.

In order to delve more deeply into the modeling results, we examine individual drift rates for each choice and each participant by entering the appropriate parameters into Equation 1 and Equation 2. The difference between the resulting drift rates for all participants' inter–temporal choice data, $\nu_N - \nu_D$, can be found in Figure 3. A highly positive difference between the now drift rate and the delayed drift rate, displayed in blue, indicates a strong preference for the now choice option. A highly negative difference between the now drift rate and the delayed drift rate, displayed in yellow, indicates a strong preference for the delayed choice option. The results show there are considerable individual differences in the extent to which participants weigh amount and delay. For example, P5 is mostly driven in their choices by the amount in dollars (as indicated by the mostly vertical transition in

[3]DIC is similar to the well–known BIC and AIC measures. However, in hierarchical models, the number of free parameters is not well–defined. As such, DIC quantifies model complexity as across–sample variability in model fit instead. Lower values of DIC indicate better support for a model from the data.

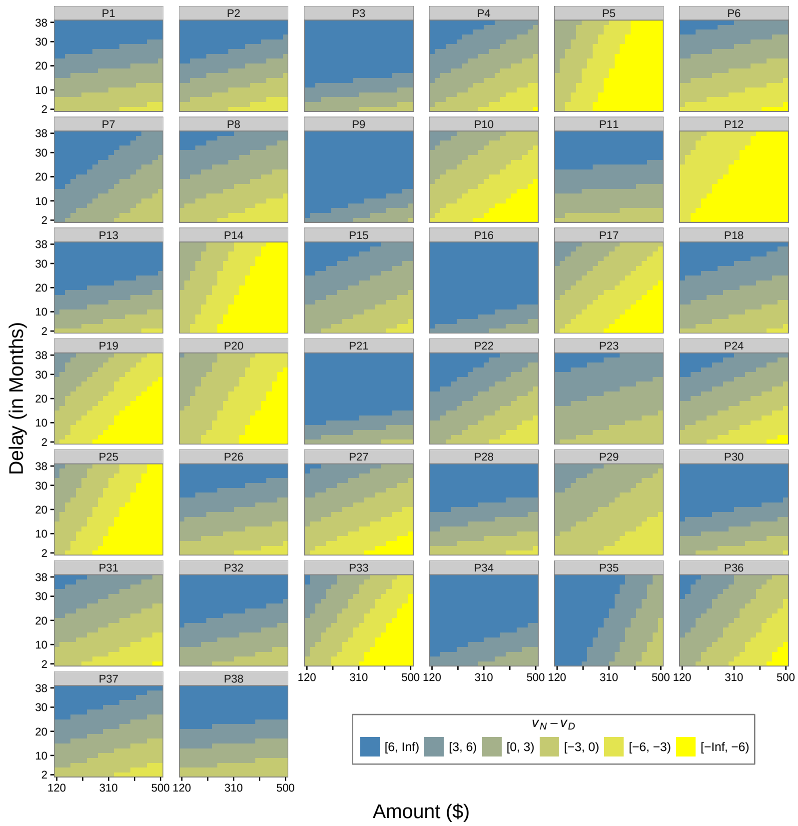

Figure 3. Absolute difference between the drift rates for the now and delayed options ($\nu_N - \nu_D$) across choices and participants (i.e., "P" panels) for the inter–temporal choice trials in Experiment 1. Positive drift rates reflect a preference for the now option and are displayed in colors that are closer to blue on the color spectrum. Negative drift rates reflect a preference for the delayed option and are displayed in colors that are closer to yellow on the color spectrum.

colors), whereas P1 is mostly driven in his choices by the delay (as indicated by the mostly horizontal transition in colors). We also see differences in the proportions of choices: participants for whom yellow dominates generally prefer the "now" option, whereas participants for whom blue dominates generally prefer the "delayed" option.

The difference between the resulting drift rates for all participants' risky choice data, $\nu_C - \nu_R$, can be found in Figure 4. Note that participants are location–matched across the figures. The results show that most participants had their strength of preference almost exclusively be determined by probability, rather than amount (the transition among colors goes predominantly along the vertical axis). There are still a few exceptions to this rule, for instance P12 who seems to weigh amount and probability almost evenly. The other stand–out observation here is that people are very risk–averse: across the board, we see a lot more blue than we see yellow.

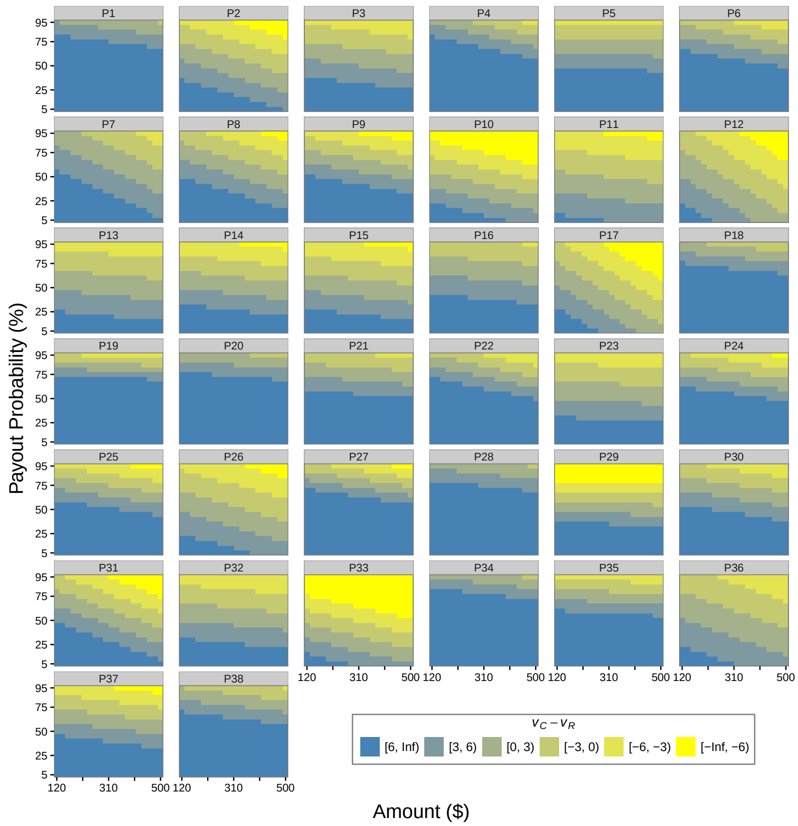

Figure 4. Absolute difference between the drift rates for the certain and risky options ($\nu_C - \nu_R$) across choices and participants (i.e., "P" panels) for the risky choice trials in Experiment 1. Positive drift rates reflect a preference for the certain option and are displayed in colors that are closer to blue on the color spectrum. Negative drift rates reflect a preference for the risky option and are displayed in colors that are closer to yellow on the color spectrum.

Discussion

Experiment 1 showed that in an inter–temporal choice setting, people prefer high payouts and short delays. In a risky choice setting, they prefer high payouts and high payout probabilities. We have showed how RT data can augment the information provided by choice responses: in conjunction with choice responses they give a measure of strength of preference. In our experiment, decisions were made quicker on average for risky than for inter–temporal choices. In addition, the formal comparison between the three variants of the LBA ("invariant", "independent", and "symmetrical") revealed that the absolute attractiveness of the now/certain choice option changes with different alternatives for the delayed/risky options as implemented by the "independent" model.

In Experiment 1 we examined the effects of delay and probability in isolation. Experiment 2 deals with options that are both delayed and probabilistic.

Experiment 2

Experiment 1 suggested that our instantiation of LBA can fit well the choice and RT data from inter–temporal and risky choices based on a simple concept of accumulated preferential strength. In Experiment 2 we examine whether our cognitive process model can provide a good explanation for choice and RT data when probability and delay combine in a single option. We also test the three different accounts of the relationship between now/certain and delayed/risky options ("invariant", "symmetrical", and "independent"). Thus, the main objective of Experiment 2 is twofold: first, to extend the LBA in order to account for the combined effect of probability and delay, and second, to examine whether the "independent" model will be the best fitting model.

In addition, Experiment 1 showed that probability and delay have a larger effect than amount on choice, and subsequently probability may be more important than delay (in the specific ranges used in our experiment). An interesting question, therefore, is whether this hypothesized ordering of relative importance, that is probability > delay > amount, would also appear when all three dimensions vary within the same option. Hence, on each trial of the experiment participants faced a choice between an option that was available now with certainty and one that differed from the fixed option in terms of probability, time of play, and amount of money. The full factorial combination of all the amounts, delays and probabilities for which we wished to elicit preferences resulted in a very large number of trials (i.e., 7,220; see Method). For this reason, in Experiment 2 we decided to collect a large amount of data from a small number (4) of participants.

Method

Participants. Four graduate students (2 female; $M_{age} = 23$) at the University of New South Wales participated in return for a $15 participation fee. In addition, they were paid $2 (i.e., outcome of the sure option) in each of 10 experimental sessions[4].

Material. The experiment consisted of a total of 7,220 inter–temporal and risky choice trials. For all choices, participants had to indicate what they preferred: $100 now for sure or $\$X$ in D months with $P\%$ certainty, with $\$X$ varying from $120 to $500 in $20 increments (for a total of 20 amounts), D varying from 2 months to 38 months in 2 month increments (for a total of 19 delays), and P varying from 5% to 95% in 5% increments (for a total of 19 probabilities). Thus, every combination of amount, delay, and probability was presented to the participant once as an alternative to $100 now for sure.

Procedure. Participants completed the experiment in 10 separate experimental sessions, each comprising of 722 choice trials. Experimental sessions were again separated by a minimum of three hours for each participant. Presentation of the sure option, and the inter–temporal and risky option on the screen was counterbalanced across participants.

Implementation of the Model

The best fit was achieved with a version of the LBA that fit the following parameters: one parameter for the upper range of starting point A, one parameter for threshold b,

[4] As in Experiment 1, participants were told that one trial from the experiment would be selected and the gamble would be played for real.

and one non–decision time parameter $t0$. As in Experiment 1, we fit three different models (independent, invariant, and symmetrical) that differed on how they model the choice process.

In Experiment 1 we observed that the same evidence accumulation (or strength of preference) process provided a good fit to both risky and inter–temporal choices. This allowed us to assume that expanding the drift rates to account for the combination of probability and delay in the same choice option would provide a good account for the risky inter–temporal choice data. The definition of the drift rates of the "independent" model for the risky inter–temporal choice task follows the same principles as for the drift rates when the two dimensions are examined in isolation, that is, a weighted sum of the attribute values of each option. Hence, we extended the model presented in Experiment 1 to account for the joint effect of delay and probability as follows:

$$\begin{aligned} \nu_{NC} &= \nu_{NC_0} - \alpha_{NC_X} \times (X/20 - 6) - \alpha_{NC_D} \times (19 - D/2) - \alpha_{NC_P} \times (P/5 - 1) \\ \nu_{DR} &= \nu_{DR_0} - \alpha_{DR_X} \times (25 - X/20) - \alpha_{DR_D} \times (D/2 - 1) - \alpha_{DR_P} \times (19 - P/5) \end{aligned} \quad (3)$$

where X, D, and P denote the amount in dollars, delay in months, and payout probability, respectively, for the delayed/risky option, ν_{NC} and ν_{DR} denote drift rates for the now/certain and the delayed/risky choice options, ν_{NC_0} and ν_{DR_0} denote offset parameters for the now/certain and the delayed/risky choice options, α_{NC_X} and α_{DR_X} denote amount scale parameters for the now/certain and the delayed/risky choice options, α_{NC_D} and α_{DR_D} denote delay scale parameters for the now/certain and the delayed/risky choice options, and α_{NC_P} and α_{DR_P} denote risk scale parameters for the now/certain and the delayed/risky choice options. $\nu_{NC} = \nu_{NC_0}$ if $X = 120$, $D = 38$, and $P\% = 5$ (the option that most favors the "now/certain" choice). $\nu_{DR} = \nu_{DR_0}$ if $X = 500$, $D = 2$, and $P\% = 95$ (the option that most favors the "delayed/risky" choice).

This results in a total of 11 parameters to be estimated: A, b, $t0$, ν_{NC_0}, α_{NC_X}, α_{NC_D}, α_{NC_P}, ν_{DR_0}, α_{DR_X}, α_{DR_D}, and α_{DR_P}. Together, these parameters should account for the distribution of response times and choice proportions for the combined inter–temporal and risky choice data.

Just as for Experiment 1, we fit two other models that test specific assumptions about the underlying choice process. The "invariant" model estimates a single ν_{NC} parameter (drift rate for the now/certain option) for all inter–temporal risky choice trials. Thus, it replaces the 4 free parameters (ν_{NC_0}, α_{NC_X}, α_{NC_D}, and α_{NC_P}) from the definition of ν_{NC} under the "independent" model (see Equation 3) with 1 free parameter. Conceptually, this simpler model assumes that the absolute value of the now/certain option does not change with different alternatives for the delayed/risky option (i.e., same absolute value for the now/certain option across all trials). The "invariant" model has 8 free parameters to be estimated. The "symmetrical" model assumes that drift rates for the now/certain option vary symmetrically (in opposite direction) with drift rates for the delayed/risky option (i.e., $\alpha_{NC_X} = \alpha_{DR_X}$, $\alpha_{NC_D} = \alpha_{DR_D}$, and $\alpha_{NC_P} = \alpha_{DR_P}$). This model has 8 free parameters to be estimated.

For Experiment 1, the "independent" model fit the data better than both the "invariant" and the "symmetrical" models. Here, we examine if the same result holds when the inter–temporal and risky elements are combined in a single choice. We used formal model comparison to find the account best supported by the data. Details on starting values, prior distributions, and number of iterations may be found in Appendix A.

Behavioral Results

Choice. All participants completed the experiment. Preference data for Experiment 2 can be found in Figure 5. The figure shows group average proportion data. Proportions close to 1, displayed in yellow, indicate a uniform preference for the delayed/risky choice. Proportions close to 0, displayed in blue, indicate a uniform preference for the $100 now/certain choice. The results show that participants prefer the now/certain option when the delayed/risky option does not pay very well (i.e., amounts not much higher than $120), when the delay is long (i.e., close to 38 months), or when the payout probability is low (i.e., close to 5%). In contrast, participants prefer the delayed/risky option when it pays well (i.e., amounts close to $500), when the delay is short (i.e., close to now), or when the payout probability is high (i.e., close to 95%).

We analyzed the data using a generalized linear mixed-effects model (Binomial distribution and logit transformation) with amount, payout probability, and delay in months of the delayed/risky option as fixed-effects predicting selection of the delayed/risky option, and random intercepts for each participant. Consistent with the observations from Experiment 1, payout probability appears to be the most important determinant of choice ($b = 3.02, z = 61.31, p. < 001$), followed by amount ($b = 1.48, z = 45.15, p < .001$) and delay ($b = -0.54, z = -20.58, p < .001$). These results indicate a positive relationship between amount and payout probability and selection of the delayed/risky option on one hand, and a negative relationship between delay and selection of the delayed risky option on the other hand. The interesting observation is that when amount, probability, and delay combine in the same choice option, amount seems to supersede delay, as it appears to have a larger effect than delay.

Response Times. Analogous to Experiment 1, the choice data indicate that people prefer high payouts, short delays, and high probabilities. RT data for Experiment 2 can be found in Figure 6. The figure shows group average RT data, binned in five equal groups. Low RTs are closer to blue on the color spectrum and high RTs are closer to yellow on the color spectrum. The results show a clear pattern: as the probability of the delayed/risky option increases, participants tend to slow down (especially once the probability exceeds .5).

We followed the same procedure as in Experiment 1 for excluding from the analysis extreme response times (2.5% of all trials were excluded). Also, we used the same statistical model to analyze the remaining trials: a generalized linear mixed-effects model (Gamma distribution and non-transformed RTs) with amount, probability and delay of the delayed/risky option, and choice (now/certain or delayed/risky) as fixed effects and participant-specific random intercepts. The analysis confirms that probability has the largest effect on RTs. Specifically, the higher the payout probability, the longer it takes for participants to respond ($b = 149.23, t = 23.62, p < .001$). The same applies for amount, though to a lesser degree ($b = 12.77, t = 2.27, p = .023$). On the other hand, the longer the delay the faster participants responded ($b = -26.21, t = -4.81, p < .001$). Also, selection of the delayed/risky option resulted in slower RTs ($b = 186.38, t = 9.70, p < .001$).

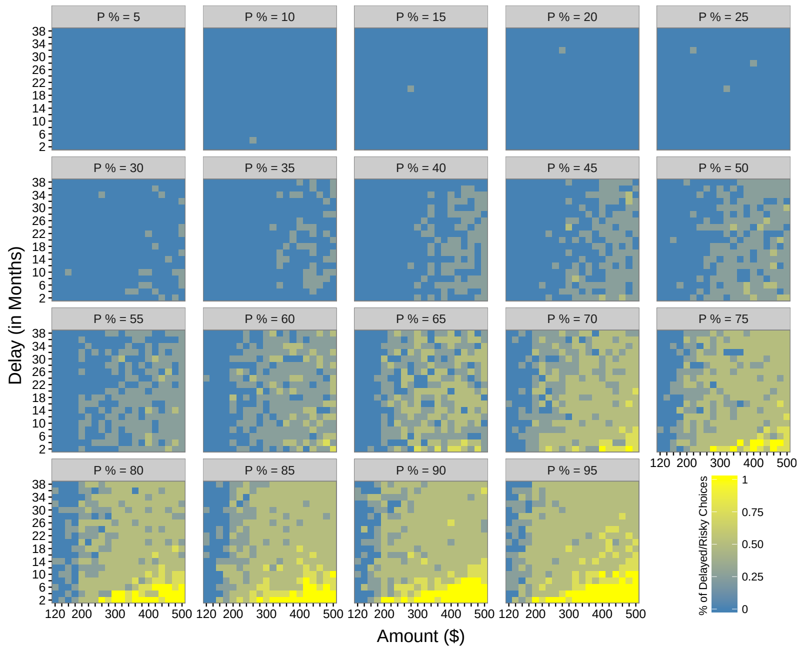

Figure 5. Aggregate choice preference data of Experiment 2. Panels represent different probability levels. A proportion of 0 (blue) indicates exclusive preference for the $100 now/certain option, a proportion of 1 (yellow) indicates exclusive preference for the delayed/risky option.

Modeling Results

Parameter convergence was satisfactory. DIC values for the three models can be found in Table 2. We obtained the same results as in Experiment 1, with the "independent" model being the best–fitting model and the "invariant" being the worst–fitting model. Consequently, this suggests that decision makers' absolute valuation of the now/certain option changes with different alternatives for the delayed/risky option.

Table 2
DICs for all three models fit to the data from Experiment 2.

Model	# Pars	DIC
Independent	11	9,419
Invariant	8	12,226
Symmetrical	8	9,612

Note: # Pars = Number of free parameters.

Moving to the estimated parameters, we examined posterior predictive data for the "independent" model, which are compared against the behavioral empirical data. The model fit the data well (i.e., choice proportions and RTs; see Figure B2 in Appendix B), only slightly underestimating response times for the slower certain/now responses, and

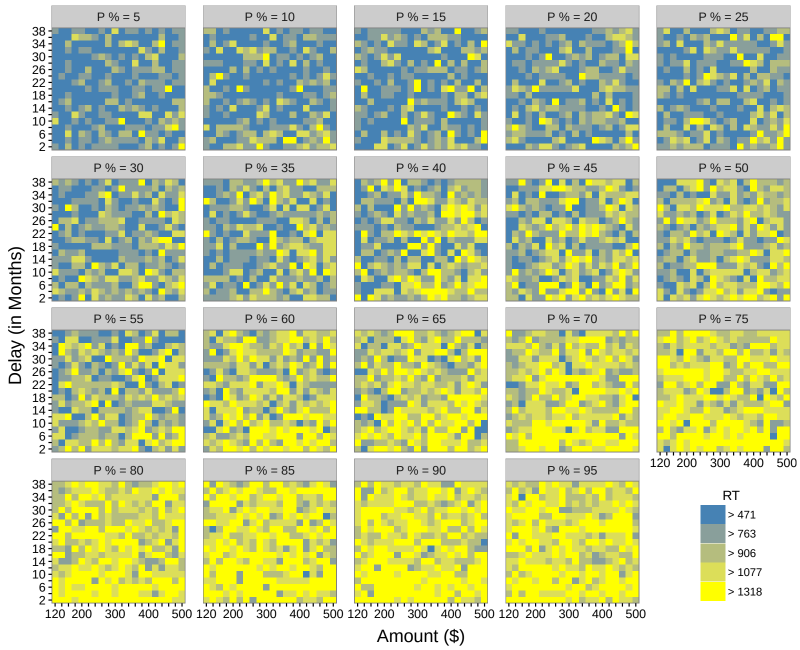

Figure 6. RT data of Experiment 2. Panels represent different probability levels. Low RTs are closer to blue on the color spectrum.

slightly overestimating response times for the delayed/risky responses. Thus, extending the LBA to account for the combined effect of time and probability and implementing the same principle of accumulated preference for inter–temporal risky choices provides a parsimonious and psychologically plausible account of choice behavior in this context.

In order to delve more deeply into this pattern, we examined individual drift rates for a representative participant by entering the appropriate parameters into Equation 3. The difference between the resulting drift rates for one participant's risky inter–temporal choice data, $\nu_{CN} - \nu_{RD}$, can be found in Figure 7. A highly positive difference between the now/certain drift rate and the delayed/risky drift rate, displayed in blue, indicates a strong preference for the now/certain choice. A highly negative difference between the now/certain drift rate and the delayed/risky drift rate, displayed in yellow, indicates a strong preference for the delayed/risky choice. Figure 7 shows that this participant's choices are almost exclusively driven by amount and payout probability as indicated by the vertical transitions in colors within and across probability levels, a pattern which is consistent with the aggregate behavioral and modeling results (see also Appendix C for individual drift rates of the remaining participants).

General Discussion

The search for understanding the principles that underlie choice in inter–temporal and risky settings has been dominated by descriptive explanations of observed behavior. Choice anomalies and deviations from EUT and DUT have led to the development of a vast number of utility-based models which propose different functional forms and extra

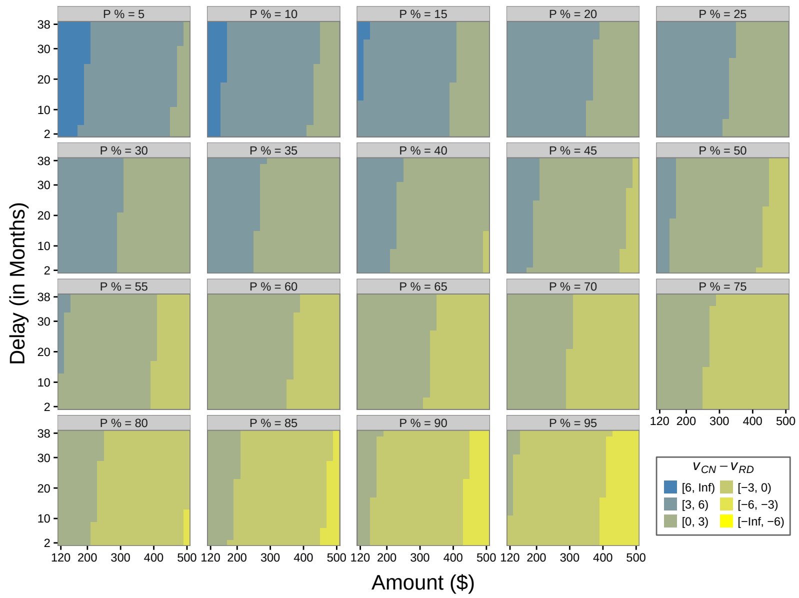

Figure 7. Absolute difference between the drift rates for the certain/now and risky/delayed options ($\nu_{CN} - \nu_{RD}$) for each choice in Experiment 2 for a representative participant (#2). Panels represent different probability levels. Positive drift rates reflect a preference for the now/certain option and are displayed in colors that are closer to blue on the color spectrum. Negative drift rates reflect a preference for the delayed/risky option and are displayed in colors that are closer to yellow on the color spectrum.

parameters in order to account for observed effects. However, in recent years, decision scientists have started to adopt cognitive processing models of choice behavior, suggesting a possible paradigm shift within judgment and decision–making research (Bhatia & Mullett, 2016; Oppenheimer & Kelso, 2015). Information processing models have been applied in many areas of decision–making and have provided psychological explanations and insights into the dynamics underlying preferential choice (see e.g., Busemeyer & Townsend, 1993; Krajbich, Armel, & Rangel, 2010; Newell & Bröder, 2008; Rodriguez et al., 2014; Trueblood et al., 2014; Usher & McClelland, 2001).

In this work, we followed a similar approach using an evidence accumulation model (LBA) to account for inter–temporal and risky choices. The novelty of our approach rests on the fact that the same modeling framework can be applied to two seemingly different types of choices, without relying on assumptions derived from either Expected Utility or Discounted Utility models. Drift rates provide a parsimonious and elegant measure for strength of preference, combining the information provided by choice responses and RTs. This combination also yielded important observations about the relative importance of each dimension (amount, probability, and delay) and their interplay in affecting participants' choices.

Choice behavior in inter–temporal and risky settings

In Experiment 1, we observed that people prefer larger, sooner to later, and certain to risky payouts. A closer inspection of the results revealed that delay and payout probability had a larger effect on choice for inter–temporal and risky gambles, respectively, as compared to amount. Comparing inter–temporal and risky choices, amount appears to matter more in an inter–temporal setting. This pattern is consistent with observations from previous research on delay and probability discounting showing that changes in amount magnitude have a larger effect in an inter-temporal than a risky choice setting (see Greenhow, Hunt, Macaskill, & Harper, 2015; Myerson, Green, Scott Hanson, Holt, & Estle, 2003; Yi et al., 2006). For the particular set of delays, probabilities, and amounts we used, a comparison of the relative importance of probability and delay across choice settings (i.e., logit regression coefficients and model parameters) indicates that probability may have a larger effect on choice compared to delay. Vanderveldt et al. (2015) found in a task where both dimensions were combined that increasing the payout probability eliminated the effect of delay, whereas when delay was increased, the effect of probability was reduced but not completely eliminated. Nonetheless, they mentioned that the superior effect of probability may be an artifact of the amounts and the range of delays and probabilities used in their study. This could also be the case in our experiment: the larger effect of probability may have been the result of the range in which we manipulated amount and delay (probability is naturally constrained between 0 and 1). With longer delays (longer than 38 months that we used in this experiment) and different starting and ending points for the range of amounts (smaller than $120 and larger than $500 that we used in this experiment), the relative importance of delay could have been different. Alternatively and consistent with our results, probability may be generally more salient than delay.

RT data showed that risky choices were made on average faster than inter–temporal choices. Participants' responses were slower when risky or delayed options were more attractive than the default option of $100 now or with certainty. In addition, clear preferences in terms of proportion produced shorter RTs. These results are suggestive of the dynamic nature of inter–temporal and risky choice, indicating that the use of static and descriptive models of choice may hinder our understanding of how preferences and choices are formed (see Dai & Busemeyer, 2014).

Experiment 2 was designed to examine the interplay between probability and delay when they are combined in a single choice option. The results largely confirmed the main trends observed in Experiment 1. Probability is the main factor that drives participants' choices, followed by amount and delay (logit regression coefficients and model parameters). A closer inspection of Figure 5 supports these results: When payout probability is less than 50%, participants predominantly chose the now/safe option. It is only when payout probability is higher than 60% that the delayed/risky option becomes more attractive and thus attains higher choice proportions. A similar interpretation of these results suggests that participants adopted a heuristic strategy based on the numerical value of probability. Since probability is the most salient dimension, participants set a criterion value for it, according to which they would start to consider the selection of the delayed/risky option. In other words, participants almost immediately discarded the delayed/risky option when probability was below 40–50%, without taking into consideration the other two attributes.

Amount and delay appear to only become relevant and play a role in the decision process when probability exceeded the predefined threshold. RT data provide further support to this interpretation: Participants slowed down when probability surpassed the assumed threshold of 40–50% (see Figure 6), possibly indicating that the extra time needed to reach a decision was associated with the processing of amount and delay. Why might this have happened? The use of such a strategy could be due to two aspects of the experimental procedure: first, the delay/risk-free option was the same throughout the task, and second, the task consisted of a large number of trials. Thus, in order to deal with such a long task, the delay/risk-free option served as the default choice and the probability of the payout indicated when to start considering the choice of the more complex and time-demanding delayed/risky option. We develop this idea in the next section by discussing the connections and similarities between heuristic and attribute-based accounts of risky and inter–temporal choice, and our cognitive process model.

LBA: An attribute-based model?

We used formal model comparison to pit three different variants of LBA against each other that differed in the assumptions they make about the absolute evaluation of the now/certain choice option. In both experiments, the "independent" model fit the data best, suggesting that the absolute attractiveness of the now/certain choice option cannot be judged in a vacuum. This goes against classic expected and discounted utility models which assume that the subjective value of an option is fixed and the product of a utility function paired with a discounting function (inter–temporal choice) or a probability weighting function (risky choice). Our cognitive process model makes no such assumptions; instead the definition of the drift rates suggests that preference for each option is formed through a weighted sum of its attributes (money, delay, and probability) and it is dependent on the numerical value of the attributes of the alternative option. In that sense, the LBA is analogous to heuristic and attribute–based models of inter–temporal and risky choice which assume that preferential choice between options is not necessarily based on underlying delay or probability discounting functions, but it is rather driven by direct comparisons between the attributes of each option (see e.g., Brandstätter et al., 2006; Cheng & González-Vallejo, 2016; González-Vallejo, 2002; Read, Frederick, & Scholten, 2013; Scholten & Read, 2010). For instance, the intertemporal choice heuristic model (ITCH; Ericson, White, Laibson, & Cohen, 2015) assumes that inter–temporal decision–making depends on the use of simple arithmetic operations or heuristics (absolute and relative comparisons of money and time) between a "now" and a "later" option. The weighted sum of these heuristics determines which option will be chosen. Weights reflect attentional focus or importance placed on each heuristic. The scaling parameters for amount, delay, and probability in the drift rates of the LBA can be conceived as serving the same purpose (see for a similar idea, Read et al., 2013). In our instantiation of the LBA, a similar result is obtained by modeling the effect of amount, delay, and probability of the delayed/risky choice on the drift rate for the unchanging now/certain option. Crucially, by contrasting this model instantiation against a model for which drift rate for the now/certain option was unaffected by the specifics of the delayed/risky choice, we were able to find numerical support for such an account. This pattern of results lends further credence to attribute-based accounts of inter-temporal and risky choice.

Our modeling analysis also adds to recent attempts that employed evidence accumulation models to account for effects in risky and inter–temporal choice. For example, Dai and Busemeyer (2014) found that an attribute–wise diffusion model, based on absolute comparisons between the dimensions of money and time, could account for three inter–temporal choice effects. Rodriguez et al. (2014) used the LBA in an inter–temporal choice setting and concluded that delayed decision–making can be also explained by sequential sampling mechanisms. Our current work extends these theoretical and practical observations and presents LBA as a model which accounts for inter–temporal and risky decision–making independently but also when the two dimensions combine in a singe choice option. The model fits the combined choice data well (see Figures B1 and B2) without incorporating trade–offs between probability and time (as is required in the probability and time trade–off model, PTT; Baucells & Heukamp, 2010, 2012), and without assuming any particular functional form for probability and delay discounting (as is required in the multiplicative hyperboloid model; Vanderveldt et al., 2015). In addition, the LBA naturally accounts for response times and implements them in the decision process as an important component of developing a preferential strength for each option. Taking all these facets together, our work presents the first attempt to model the combined effect of probability and delay through an evidence accumulation process, and to provide psychological explanations about preferential choice that rely on the simultaneous examination of choice and RT data.

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Appendix A
Distributional Choices

Experiment 1

Starting values for the MCMC chains for individual parameters were drawn from the following distributions: $b_D \sim N(2, 0.2)|(0, \infty)$, $b_R \sim N(1.5, 0.15)|(0, \infty)$, both As $\sim N(2, 0.2)|(0, \infty)$, $\nu_{N_0} \sim N(4.5, 0.45)|(0, \infty)$, $\nu_{D_0} \sim N(4.5, 0.45)|(0, \infty)$, $\nu_{C_0} \sim N(6, 0.6)|(0, \infty)$, $\nu_{R_0} \sim N(4.5, 0.45)|(0, \infty)$, $\alpha_{N_X} \sim N(0.11, 0.011)|(0, \infty)$, $\alpha_{N_D} \sim N(0.17, 0.017)|(0, \infty)$, $\alpha_{D_X} \sim N(0.15, 0.015)|(0, \infty)$, $\alpha_{D_D} \sim N(0.3, 0.03)|(0, \infty)$, $\alpha_{C_X} \sim N(0.06, 0.006)|(0, \infty)$, $\alpha_{C_R} \sim N(0.25, 0.025)|(0, \infty)$, $\alpha_{R_X} \sim N(0.09, 0.009)|(0, \infty)$, $\alpha_{R_R} \sim N(0.4, 0.04)|(0, \infty)$, and $t0 \sim N(0.2, 0.02)|(0, \infty)$. The notation $\sim N(,)$ indicates that values were drawn from a normal distribution with mean and standard deviation parameters given by the first and second number between parentheses, respectively. The notation $|(,)$ indicates that the values sampled from the normal distribution were truncated between the first and second numbers in parentheses.

The hierarchical set–up prescribes that all individual parameters come from a truncated Gaussian group–level distribution (truncated to positive values only). Thus, for each parameter to be estimated, we are estimating a group level mean parameter and a group level standard deviation parameter. All group level mean parameters are normally distributed, both b_μs $\sim N(2, 1)|(0, \infty)$, both A_μs $\sim N(2, 1)|(0, \infty)$, $\nu_{\mu-N_0} \sim N(4.5, 2)|(0, \infty)$, $\nu_{\mu-D_0} \sim N(4.5, 2)|(0, \infty)$, $\nu_{\mu-C_0} \sim N(4.5, 2)|(0, \infty)$, $\nu_{\mu-R_0} \sim N(4.5, 2)|(0, \infty)$, $\alpha_{\mu-N_X} \sim N(0.1, 0.05)|(0, \infty)$, $\alpha_{\mu-N_D} \sim N(0.3, 0.15)|(0, \infty)$, $\alpha_{\mu-D_X} \sim N(0.1, 0.05)|(0, \infty)$, $\alpha_{\mu-D_D} \sim N(0.3, 0.15)|(0, \infty)$, $\alpha_{\mu-C_X} \sim N(0.1, 0.05)|(0, \infty)$, $\alpha_{\mu-C_R} \sim N(0.3, 0.15)|(0, \infty)$, $\alpha_{\mu-R_X} \sim N(0.1, 0.05)|(0, \infty)$, $\alpha_{\mu-R_R} \sim N(0.3, 0.15)|(0, \infty)$, and $t0_\mu \sim N(0.2, 0.07)|(0, \infty)$. All group level standard deviation parameters are gamma distributed, with a shape and a scale parameter of 1. Starting values for the MCMC chains for group level μ parameters were drawn from the same distributions as those for the individual parameters, and starting values for group level σ parameters were derived from starting value distributions for the individual parameters by dividing the mean by 10 and the standard deviation by 2. These prior settings are quite uninformative, and are based on previous experience with parameter estimation for the LBA model. As a result, the specific settings will not have a large influence on the shape of the posterior. For more details on distributional choices for the priors, we refer the reader to Turner et al. (2013).

For sampling, we used 32 interacting Markov chains and ran each for 1,000 burn-in iterations followed by 1,000 iterations after convergence. The two tuning parameters of the differential evolution proposal algorithm were set to standard values used in previous work: random perturbations were added to all proposals drawn uniformly from the interval [-.001, .001]; and the scale of the difference added for proposal generation was set to $\gamma = 2.38 \times (2K)^{-0.5}$, where K is the number of parameters per participant. The MCMC chains blocked proposals separately for each participant's parameters, and also blocked the group–level parameters in $\{\mu, \sigma\}$ pairs.

Experiment 2

Starting values for the MCMC chains for individual parameters were drawn from the following distributions: $b \sim N(2, 0.2)|(0, \infty)$, $A \sim N(2, 0.2)|(0, \infty)$, $\nu_{NC_0} \sim$

$N(6, 0.6)|(0, \infty)$, $\nu_{DR_0} \sim N(6, 0.6)|(0, \infty)$, $\alpha_{NC_X} \sim N(0.1, 0.01)|(0, \infty)$, $\alpha_{NC_D} \sim N(0.15, 0.015)|(0, \infty)$, $\alpha_{NC_R} \sim N(0.2, 0.02)|(0, \infty)$, $\alpha_{DR_X} \sim N(0.1, 0.01)|(0, \infty)$, $\alpha_{DR_D} \sim N(0.15, 0.015)|(0, \infty)$, $\alpha_{DR_R} \sim N(0.2, 0.02)|(0, \infty)$, and $t0 \sim N(0.2, 0.02)|(0, \infty)$.

All group level mean parameters are normally distributed, $b_\mu \sim N(2, 1)|(0, \infty)$, $A_\mu \sim N(2, 1)|(0, \infty)$, $\nu_{\mu-NC_0} \sim N(6, 2)|(0, \infty)$, $\nu_{\mu-DR_0} \sim N(6, 2)|(0, \infty)$, $\alpha_{\mu-NC_X} \sim N(0.1, 0.05)|(0, \infty)$, $\alpha_{\mu-NC_D} \sim N(0.15, 0.075)|(0, \infty)$, $\alpha_{\mu-NC_R} \sim N(0.2, 0.1)|(0, \infty)$, $\alpha_{\mu-DR_X} \sim N(0.1, 0.05)|(0, \infty)$, $\alpha_{\mu-DR_D} \sim N(0.15, 0.075)|(0, \infty)$, $\alpha_{\mu-DR_R} \sim N(0.2, 0.1)|(0, \infty)$, and $t0_\mu \sim N(0.2, 0.07)|(0, \infty)$. All group level standard deviation parameters are gamma distributed, with a shape and a scale parameter of 1. Starting values for the MCMC chains for group level μ parameters were drawn from the same distributions as those for the individual parameters, and starting values for group level σ parameters were derived from starting value distributions for the individual parameters by dividing the mean by 10 and the standard deviation by 2.

For sampling, we used 32 interacting Markov chains and ran each for 1,000 burn-in iterations followed by 1,000 iterations after convergence. The two tuning parameters of the differential evolution proposal algorithm were set to standard values used in previous work: random perturbations were added to all proposals drawn uniformly from the interval [-.001, .001]; and the scale of the difference added for proposal generation was set to $\gamma = 2.38 \times (2K)^{-0.5}$, where K is the number of parameters per participant. The MCMC chains blocked proposals separately for each participant's parameters, and also blocked the group–level parameters in $\{\mu, \sigma\}$ pairs.

Appendix B
Posterior Predictives

Posterior predictives for the “independent” model for Experiment 1 can be found in Figure B1.

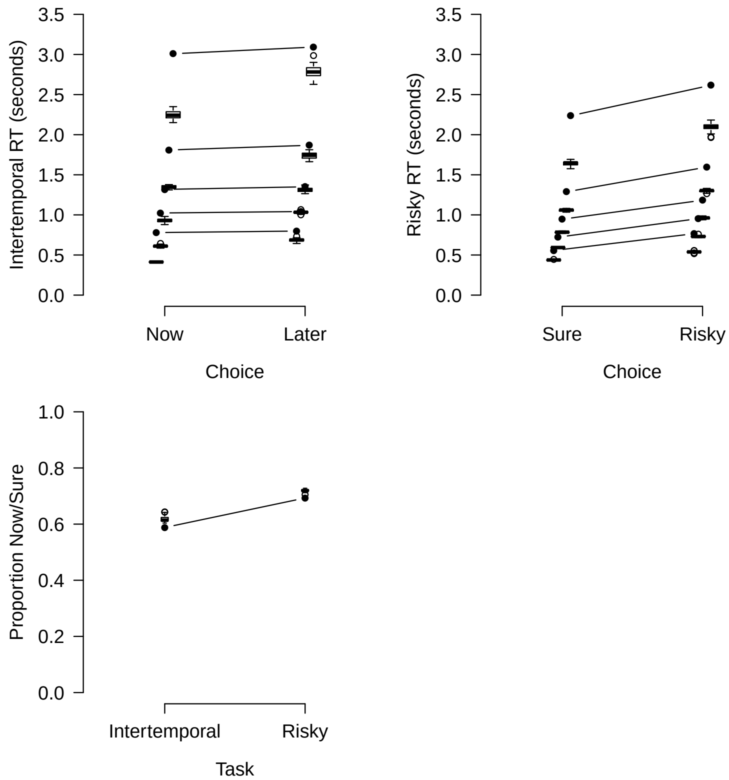

Figure B1. Posterior predictives for Experiment 1. For RTs, .1, .3, .5, .7, and .9 deciles (y–axis) calculated for intertemporal RTs (top–left panel) and risky RTs (top–right panel), both separated for choice type, and proportion of now/sure responses (bottom–left panel). For all panels, box–and–whiskers represent posterior predictive data (the box contains 50% of the simulated data, with a bar across the middle indicating the median, and whiskers extend to the data extremes) and lines represent data.

Posterior predictives for the “independent” model for Experiment 2 can be found in Figure B2.

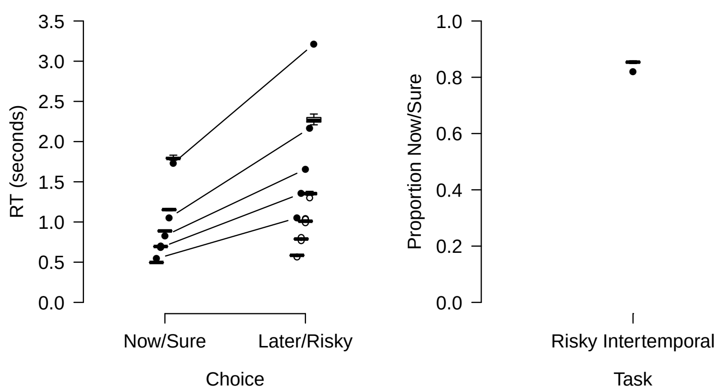

Figure B2. Posterior predictives for Experiment 2. For RTs, .1, .3, .5, .7, and .9 deciles (y–axis) calculated for intertemporal risky RTs (left panel), separated for choice type, and proportion of now/sure responses (right panel). For both panels, box–and–whiskers represent posterior predictive data (the box contains 50% of the simulated data, with a bar across the middle indicating the median, and whiskers extend to the data extremes) and lines represent data.

Appendix C

Invididual drift rates in Experiment 2

Individual drift rates for each choice in Experiment 2 for participants 1, 3 and 4.

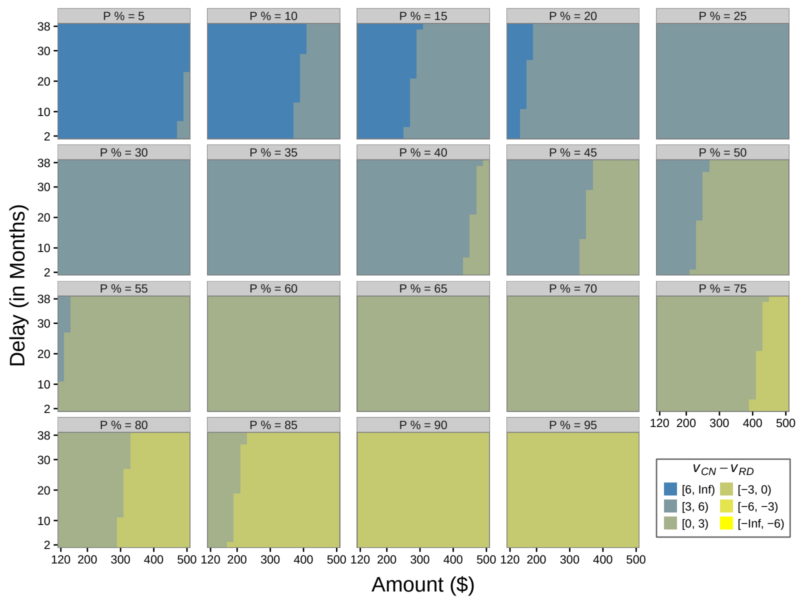

Figure C1. Participant 1

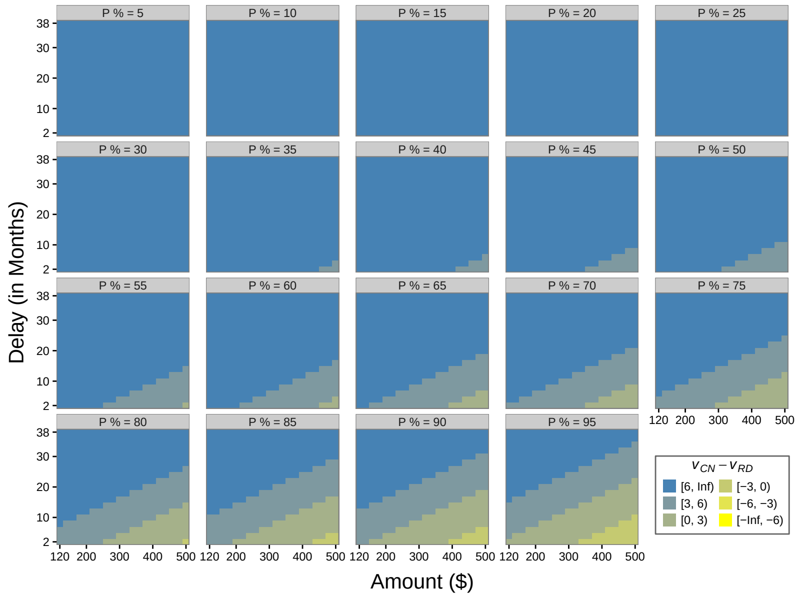

Figure C2. Participant 3

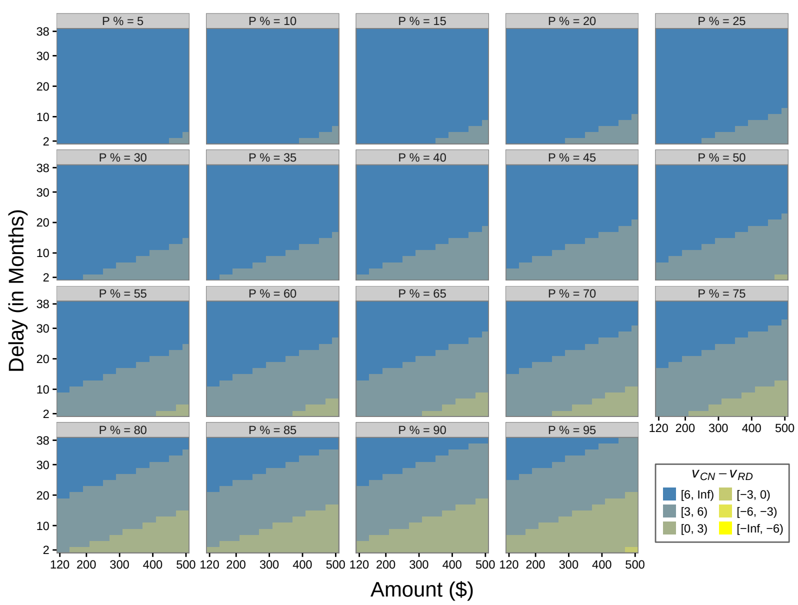

Figure C3. Participant 4