but we think this is unlikely, simply because the situations to which such tools are applicable (e.g., data follow a quantum Gaussian distribution) do not arise very often when one is analyzing real data. Until statistical tools based on QP find a place in everyday data analysis, we remain unconvinced that QP makes sense as a normative account of everyday inference.

Regarding the algorithmic level, we think that P&B are on more solid ground: there is some justification for thinking about QP models as mechanistic accounts. Consider the model used to account for Shafir and Tversky's (1992) data on the prisoner's dilemma. It relies on an interference effect to account for the fact that participants defect whenever the opponent's action is known but cooperate when it is unknown. This interference does not emerge as part of an optimal solution to the inference problem given to the decision maker, nor is it characterized at a neural level. It is clearly intended to refer to a psychological mechanism of some kind.

In view of this, a mechanistic view of QP seems to provide the right way forward, but at times it is difficult to understand what the mechanisms actually are. To take a simple example, why are some questions incompatible and others are compatible? P&B suggest that "[a] heuristic guide of whether some questions should be considered compatible or not is whether clarifying one is expected to interfere with the evaluation of the other" (sect. 2.2). This seems sensible, but it begs the question. One is naturally led to ask why some psychological states interfere and others do not. This is difficult to answer because the QP formalism is silent on how its central constructs (e.g., interference) map onto psychological mechanisms. In our own work (Fuss & Navarro, in press) we have explored this issue in regards to the dynamic equations that describe how quantum states change over time. Specifically, we have sought to describe how these equations could arise from mechanistic processes, but our solution is specific to a particular class of models and we do not claim to have solved the problem in general. In our view, understanding how formalisms map onto mechanisms is one of the biggest open questions within the QP framework.

In short, we think that the potential in QP lies in developing sensible, interpretable psychological mechanisms that can account for the otherwise puzzling inconsistencies in human decision making. It might be that human cognition cannot be described using the standard provided by classical probability theory, but turns out to be more consistent with QP theory. That doesn't make QP a good tool for rational analysis, but it would make it an interesting psychological mechanism, particularly if it is possible to provide clear and consistent interpretations for its central constructs. Should events unfold in this way, then statistics would continue to rely on classical probability for its theoretical foundation, but cognitive modelers could use quantum probability in many instances. There is nothing incompatible about these two states.

A quantum of truth? Querying the alternative benchmark for human cognition

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Abstract: We focus on two issues: (1) an unusual, counterintuitive prediction that quantum probability (QP) theory appears to make regarding multiple sequential judgments, and (2) the extent to which QP is an appropriate and comprehensive benchmark for assessing judgment. These issues highlight how QP theory can fall prey to the same problems of arbitrariness that Pothos & Busemeyer (P&B) discuss as plaguing other models.

1. Multiple sequential judgments. One of the basic tenets of quantum probability (QP) is that the order in which questions are asked of a person will affect how he or she feels about the answer. Pothos & Busemeyer (P&B) illustrate this sequential nature of QP using the Clinton/Gore attitude assimilation effect reported by Moore (2002). The key result is that the percentage of participants endorsing Clinton as honest *increases* by 7% when Clinton is rated after Gore, but Gore's honesty endorsement *decreases* by 8% when he is asked about after Clinton. Thus the politicians become more similar (assimilate) when they are asked about second (3% difference in endorsement rates) than when asked about first (18% difference in endorsement rate). This point is illustrated in Figure 3 of P&B, reprinted here as the top-left panel of Figure 1.

P&B show that if the initial state vector is projected onto the |Gore yes> basis vector first, followed by the |Clinton yes> basis vector, Clinton will be judged as more honest than if the initial state vector is projected onto |Clinton yes> directly. Thus, the authors explain how asking about the honesty of Gore first, will lead to a subsequently more positive judgment of Clinton's honesty.

An unusual prediction that follows is that as these projections continue, the state vector will gravitate toward the zero point. As an illustration, consider the effect of asking successive questions about the honesty of additional presidents. We assume that subsequent questions have representations as basis vectors in the outcome space. Just as the state vector from |Gore Yes> is projected onto |Clinton Yes>, we assume that subsequent questions cause the state vector to project onto the next appropriate basis vector. As shown in Figure 1, as each state vector projects onto the nearest point of the next basis vector, subsequent state vectors will get shorter (by definition).

Although we agree that asking about the honesty of a number of politicians might put one in a progressively more suspicious frame of mind, it seems unlikely that the believability of any president should necessarily decrease (reaching close to zero in as few as 10 questions) as more questions are asked. Imagine, for example, if the sixth president was Lincoln or Washington.

A possible solution to this problem is to assume that the state vector somehow resets or recalibrates itself, perhaps because of a decay of the effect of initial questions (i.e., forgetting). P&B argue that one of the benefits of QP is that it is based on axiomatic principles, thus avoiding problems of "arbitrariness" common in other explanatory frameworks (e.g., heuristics). Adding a "recalibration" step would appear to be a post-hoc fix outside of the main principles, and as such, something that P&B are at pains to avoid. This example highlights why formal frameworks make such attractive theoretical tools: they make strong, testable predictions.

2. An appropriate benchmark? Two criteria have been prominent in the search for an appropriate benchmark for probability judgment: correspondence and coherence (eg., Hammond 1996). These terms, stemming from philosophy, invite different ways of assessing truth: via correspondence with observable facts, and via having a set of internally consistent (coherent) beliefs. Several commentators have argued that both criteria need to be considered for adequate assessment of judgments (e.g., Dunwoody 2009; Newell 2013).

P&B argue strongly that coherence should be assessed against the axioms of QP not CP – hence allowing Linda to be more likely a feminist bank teller than just a bank teller – but what of correspondence? Consider the correspondence error that homicide is judged the more likely cause of death than suicide (e.g., Lichtenstein et al. 1978). Such a judgment is an error because it

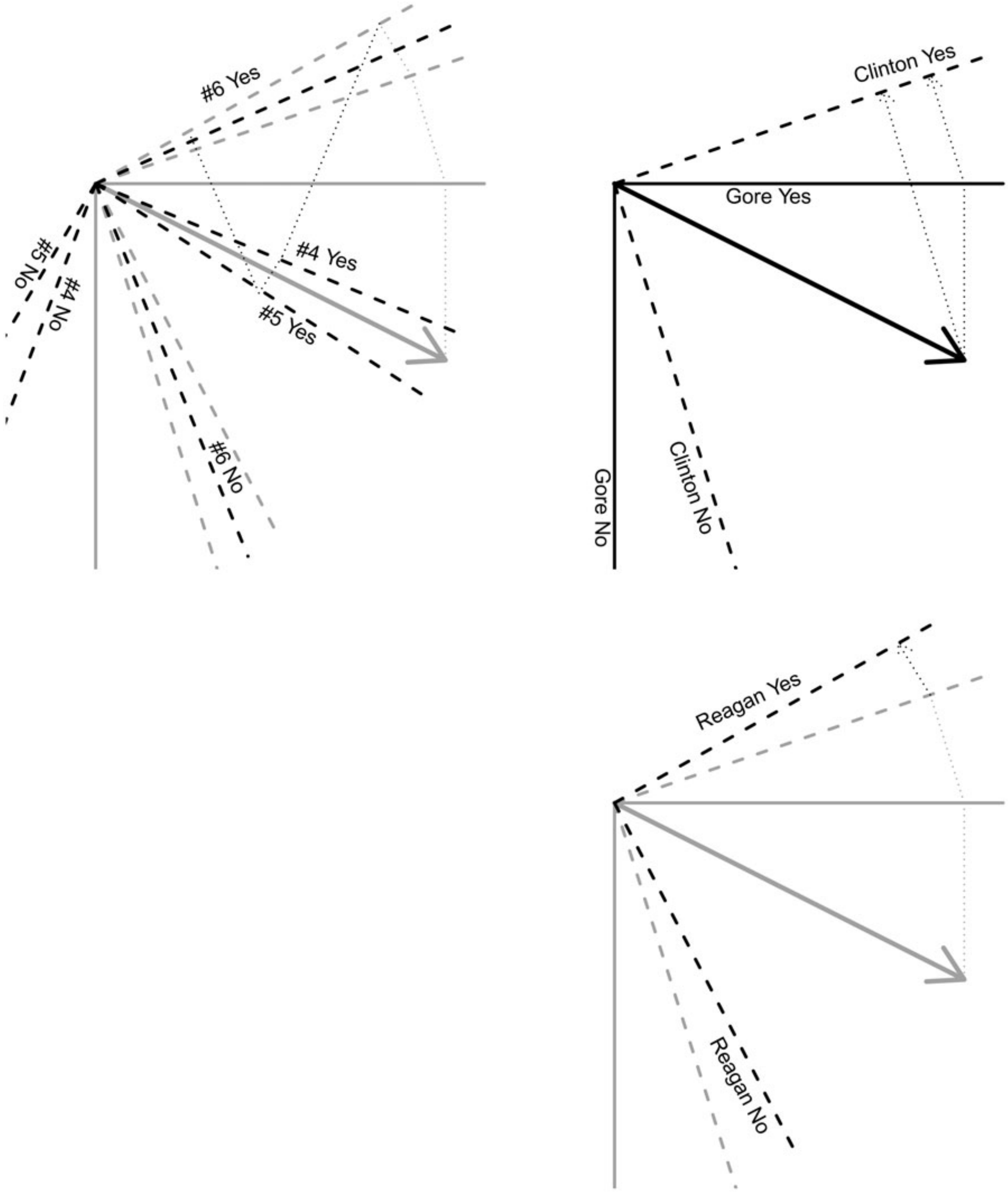

Figure 1 (Newell et al.). Multiple sequential judgments lead to a belief state that comes ever closer to zero. See text for details.

does not correspond with the fact that there are more suicides per capita than homicides. Such an "irrational" judgment emerges from the same cognitive system as the Linda judgment and, therefore, should, according to P&B's thesis, be explicable in the QP framework. Our intuition is that QP theory would explain this effect by constructing bases corresponding to representations of death from suicide, death not from suicide, death from homicide, and death not from homicide (in much the same way as bases are constructed for happy and ~happy in P&B's Fig. 1). It might be assumed that people's initial state vector, because of something akin to "availability," is closer to the homicide basis vector than the suicide vector. This would lead to a larger projection, and, therefore, a judgment of higher probability of homicide than suicide.

Assuming that it is possible to construct such a space, one may ask what predictions QP theory would make were we to ask the participants to sequentially judge the likelihood of both suicide and homicide. To generate such predictions, however, we must first know whether, for example, the two questions are compatible. We must also know whether the initial vector lies between the homicide and suicide basis vectors, or between the homicide and not suicide vectors, for example. Such decisions about the parameters of the model influence the qualitative pattern that QP theory will produce, for example, that compatibility will determine whether we expect the judgments to be invariant to the order of the questions. Similarly, the location of the initial state vector, for incompatible questions, will determine whether the second judgment increases or decreases relative to when it was judged first. Although not relevant to the current example, the principles of entanglement and superposition have similar effects on the qualitative pattern that QP theory predicts.

To call the decisions about such principles in QP theory "arbitrary" may be going too far – P&B provide intuition for when we might expect some of these principles to hold (e.g., compatibility). However, we argue that an understanding of these unique aspects of QP theory, to the point that they are predictable, is a major issue that needs addressing before QP theory can vie to be the framework of choice.

Quantum modeling of common sense

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Abstract: Quantum theory is a powerful framework for probabilistic modeling of cognition. Strong empirical evidence suggests the context- and order-dependent representation of human judgment and decision-making processes, which falls beyond the scope of classical Bayesian probability theories. However, considering behavior as the output of underlying neurobiological processes, a fundamental question remains unanswered: Is cognition a probabilistic process at all?

Using quantum theory for understanding cognitive processes was mainly inspired by the Copenhagen interpretation of quantum mechanics in the 1920s. However, it took scientists almost a century to formalize cognitive models utilizing the unique features of quantum probability (Aerts & Aerts 1995).

Based on the Kolmogorov probability axioms, classical theories rely on intuitive mathematical foundations that have substantiated their application for modeling psychological phenomena for a long time. Nevertheless, a large body of evidence on order and context dependent effects and the violation of the law of total probability suggest that human judgment might follow counterintuitive principles.

One of the major achievements of quantum theory was the conceptualization of the superposition principle. Measurements of a system, which is linearly composed by a set of independent states, assign it to a particular state and stop it from being in any other one. As a consequence, the act of observation influences the state of a phenomenon being observed, and indicates a general dependency on the order of observations. These are two significant aspects of cognitive processes that dramatically challenge the classical theories.

The lack of reliable mathematical formalisms within the classical frameworks to address such issues, the inclusion of the classical assessments as specific (trivial) quantum assessments, and the wider application range of quantum probability, even suggest the superiority of quantum theory for modeling cognition.

Pothos & Busemeyer (P&B) discuss this matter thoroughly and elegantly. Modeling non-commutative processes is a unique characteristic of quantum theory, which provides a noticeable distinction between compatible and incompatible questions for cognitive systems. From the psychological point of view, incompatibility between questions refers to the inability of a cognitive agent in formulating single thoughts for combinations of corresponding outcomes. Each question influences the human state of mind in a context-dependent manner and, therefore, affects the consideration of any subsequent question (induced order dependency). Bayesian models simply fail to represent the context- and order-dependent scenarios, whereas for non-dynamic compatible questions their predictions converge to the assessment of the quantum probability theories.

In addition to the more general nature of quantum theory in describing static cognitive processes, this formalism could also be viewed as an extension of the classical probabilities for dynamic processes. Whereas time evolution is in both theories represented as linear transformations, the dynamic quantum probabilities are nonlinear functions, which in general implicate possible violations of the law of total probability.

In the long run, the framework of quantum modeling of cognitive processes presents a generalization of the Bayesian theories, with a deeper notion of uncertainty and natural approaches to problems.

Despite the convincing superiority of quantum theories with respect to classical probability models, a fundamental question is still open: Are cognitive processes governed by stochastic principles at all? To address this question, we will focus on the compatibility and expediency of stochastic approaches from the point of view of neuroscience research, and omit any involvement in philosophical discussions on the nature of human judgment and decision making.

Following Griffiths and colleagues (2010), the authors characterize probabilistic models of cognition as "top-down" or "function-first." Furthermore, P&B espouse the philosophy that "neuroscience methods and computational bottom-up approaches are typically unable to provide much insight into the fundamental why and how questions of cognitive process" (sect. 1.2) and hence suggest the *right* modeling strategy as beginning with abstract (stochastic) principles and then reducing them into (deterministic) neural processes.

However, the assumption of a stochastic nature of cognition and behavior has severe consequences both mathematically and biologically.

1. *Convergence of stochastic and deterministic results:* Using a deterministic dynamic systems model, computational neuroscience and biophysics improved our understanding of neuronal processes in the last decades. Particularly, the models reproduced and predicted neurochemical and electrophysiological processes that were shown to induce alterations in the behavior of animals and human (Knowlton et al. 2012; Maia & Frank 2011; Noori & Jäger 2010). The sum of these theoretical models and their experimental validations suggests a deterministic relationship between neural processes and behavior. On the other hand, stochastic models of cognition assign to each behavioral output a proper random variable in a probability space. Therefore, a certain behavior could be characterized as a deterministic function of biological variations and a random variable simultaneously. This paradoxical duality requires the deterministic and stochastic functions to converge to the same behavioral outcome under given conditions. Consequently, the compatibility of the top-down stochastic approaches with biological findings and the possibility of a reduction into lower-level neural processes depend on the existence of appropriate convergence criteria, which have not been provided to date.

2. *Interactions of neural processes and human behavior:* From cellular dynamics to oscillations at the neurocircuitry level, numerous studies have identified biological processes that define/influence behavior (Morrison & Baxter 2012; Noori et al. 2012; Shin & Liberzon 2010). Therefore, cognition is a causal consequence of a series of biological events. In light of these investigations, a cognitive process of a stochastic nature inherited its "random" character from its underlying biology. In other words, the top-down reduction of the abstract stochastic principles into neural processes implies probabilistic dynamic behavior of neural systems at different spatiotemporal scales. However, the lack of theoretical or experimental models confirming the probabilistic nature of neural mechanisms challenges the proposed top-down strategy.

In conclusion, although the quantum probability theories significantly extend the application field of classical Bayesian theories for modeling cognitive processes, they do not address the general criticisms towards top-down modeling approaches.